

**Galton's Quincunx: Creating an Accessible Model of the Central Limit Theorem**

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### Abstract

Mathematical models are visuals which help students understand difficult concepts. The quincunx or Galton board is an example of one created by Sir Francis Galton which showcases the central limit theorem through the distribution of beads between cells appearing bell-shaped. In this paper, the process of creating and sharing a Galton board design for the classroom was outlined. It was previously found that the peg distance to bead size ratio and peg shape affects the distribution of beads in a Galton board (Chong, 2021; Seymour, 1996). Therefore, two prototypes with different peg shapes were created each with a set peg distance, so while testing, bead size could be altered to find the best ratio. A few previous designs were found in literature which informed the alpha designs for each prototype (Business Performance Improvement, n.d.; Index Fund Advisors, 2024). After each alpha prototype was created, it was tested and iterated until it worked. The two prototypes were then compared to each other; the design featuring hexagonal pegs presented a more normal distribution. Thus, the design's STL file and a guide to building a Galton board was added to a website posted online for teachers. Since the design created was easily accessible, printable, and accurate in portraying a normal distribution, it was deemed successful. This design can be used by teachers beyond statistics to provide a visual of the central limit theorem as well as contribute to the small communities of literature surrounding the Galton board and other mathematical models.

*Keywords:* Galton board, central limit theorem, normal distribution, three-dimensional design

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### **Galton's Quincunx: Creating and Accessible Model of the Central Limit Theorem**

In the past few years, teaching has seen a switch as a new style of teaching relying heavily on physical models has arisen. This shift can be dated back to the 1980s when a complete curriculum redesign occurred in Singapore. The government implemented a requirement for more visual mathematical models in the classroom resulting in a steep increase in performance in international math competitions (Lisciandrello, n.d.). With a rise in availability of 3D printing, the effects have recently become widespread into other classrooms around the world. This is exemplified through the increase in teacher made mathematical models such as the fraction and decimal tools (Greenstein & Fernandez, 2023).

#### **The Galton Board**

The method of showing something visually instead of explaining it verbally was pioneered by Sir Francis Galton who in 1876 tried to present his findings on the distribution of univariate data ("Galton, Francis", 2020). However, he found difficulty in explaining his results on the normal distribution, a new concept, to those around him because they were uninformed. To combat this issue, Galton set out to show it visually. He did so by creating a mechanism from a Bagatelle board that would visually showcase the creation of a normal distribution ("Galton, Francis, 2020; Pritchard, 2006). Beads were placed at the top of the board in the bead chamber. Then the floor of the chamber was released allowing the beads to flow down the board. On their way down the board, they would hit individual pegs causing them to change direction randomly and collect in cells at the bottom of the board. When created and used correctly the distribution of beads between the cells would model a normal distribution (Seymour, 1996).

### **Normal Distributions**

A normal distribution also known as a bell curve describes the shape of a univariate data set when it is symmetric, bell shaped, and has a peak around the mean of the data set (“Measures of Shape”, n.d.). The central limit theorem is a statistical theorem which states that if there is a sufficiently large distribution of random mean sample statistics, the distribution will be approximately normal (LaMorte, 2016). In a Galton board, as the beads hit each peg their directions randomly change. Even though it is seemingly random, there is a pattern. Because of the equal chances of a bead going left or right when it hits a peg, the beads have a higher probability of landing in a middle cell following the shape of a normal distribution. In terms of the central limit theorem, each bead represents a mean sample statistic and the whole batch of beads represents a random sample of mean sample statistics. If a sufficiently large number of beads or random sample statistics is utilized in the board, the beads will be normally distributed (Galvin, n.d.).

### **The Galton Board in the Classroom**

3D models in the classroom have seen a great increase in usage due to their appeal to teachers for helping students understand content. In the field of anatomy, a study was conducted to observe a difference between understanding of content with and without 3D models, and it was found that those who utilized 3D models scored higher on a post test, and were more satisfied with their education (Ye et. al, 2020). Although studies have not been conducted into the usefulness of 3D models in statistics, a similar outcome can be theorized. Therefore, Galton boards could be a great asset to statistics teachers (Seymour, 1996).

### **Rationale**

Since mathematical models are shown to improve classroom understanding (Greenstein & Fernandez, 2023), this project aimed to create an accessible and easy to print Galton board for teachers as well as contribute to the growing community of teacher-used mathematical models found in current literature.

Online, there were Galton boards available, but they were either inaccurate or costly. Many of them didn't come with printing and assembly instructions meaning teachers who were unfamiliar with 3D printing risked causing printing failures (Aranda et al., 2022). These failures could create a new variable which affected the distribution of beads making it not normal (Galvin, n.d.). When they came pre-assembled and worked accurately, they weren't free. Therefore, the improvement of free online Galton board designs from this project greatly benefited teachers.

Further, this project benefited me as it provided me with the opportunity to gain more experience with 3D modeling and printing. For 2 seasons, I participated on one of my school's robotics teams where I always wanted to help design and 3D model the robot, yet I never had the chance due to my lack of knowledge in this field. With this project, I was fully committed to developing this skill.

Additionally, this project was directly related to my internship since I worked at KSU's MAD Lab, a 3D printing lab, where I had access to different types of 3D printers. On a day to day basis, every lab partner modeled and printed items to aid in simple tasks. The Galton board was an example of this. In addition, my mentors Dr. Dominic Thomas and Dr. Harron were industry experts in 3D printing and did their own 3D design research who provided me with extra resources.

### **Statement of Focus and Sub Problems**

The focus of this project was to create an accessible and easily printable Galton board design to be shared with teachers. The accessibility of this design was defined by how many teachers could actually utilize it; the more teachers there were that could use it, the more accessible the design was considered. Therefore, the final Galton board model was made available on a published website alongside a guide for printing and assembly, so even teachers who were unfamiliar with 3D printing could use the design. Utilizing current literature and resources available at the KSU Modular Agile Deployment lab, this project addressed the following sub problems:

- Basic Sub Problem: What skews the distribution of beads in a Galton board?
- Basic Sub Problem: What working Galton board designs are prevalent in literature?
- Design Sub Problem: Fabricate a shareable Galton Board design for teachers which showcases the Central Limit Theorem
  - Measure of Success: A successful design will be easily accessible and printable, accurate in portraying a normal distribution, and approved by Dr. Thomas or Dr. Harron



### **Literature Review**

Galton boards can be used in the classroom by teachers to help students visually understand the central limit theorem. Therefore, the overarching goal of the project was to create a design which teachers can use for this purpose. The first task in completing this goal was to consult previous literature to inform the fabrication of the designs. Specifically, literature would establish what features created a skew of the distribution of beads in a Galton board as well as what working designs already existed.

### **Skewed Distributions**

Galton boards are immensely theory based. In real-life application, they almost never perfectly match a normal distribution because creating a consistent random change of direction from the pegs is nearly impossible (Oshman, 2002). Thus, to create an approximately accurate model, it was important to know the factors that skew the distribution of beads in a Galton board, so they could be avoided when designing the model.

Previous literature suggested that one design factor that plays into skewing the distribution is the distance between the pegs of the board. It was found that as the distance between pegs increased, the number of beads which accumulate in the middle cells increased (Chong, 2021). An additional design factor was found to be peg shape. In Galton boards, there are two common peg designs: circular and hexagonal. Circular pegs were a part of the original design by Galton, and they acted as obstacles to the balls as they fell through the board (Mulvhill, n.d.). Hexagonal pegs are an optimization of the board which created channels for the balls to pass through (Seymour, 1996). If designed properly, both can be effective (Chong, 2021).

With this literature in mind, a few features were implemented into all designs. First of all, the distance between the pegs of the board were set to be larger than the goal bead size. That

being the bead size that the board design is optimized for. This distance would not change between versions of the same prototype. Instead to optimize the peg distance to bead size ratio, during the testing stage, the bead size would be changed if the distribution wasn't normal. Secondly, since neither peg design is shown to be better in literature, two board designs would be designed and iterated independently from each other, one for each peg shape.

### **Effective Commercial Designs**

When fabricating a Galton board, it was key to know what working designs existed because the features on the designs in literature could inform the alpha designs of each prototype. Before finding these designs, it was important to define a working Galton board. A working model was a model that actually approximated a normal distribution and smoothly ran. There were two examples of these in literature, one for each peg shape. The first example of a working design featured 6000 1 mm steel balls. Additionally, it had 13 rows of hexagonal pegs placed in a triangle formation ("Galton Board with Pascal's Triangle", n.d.; Index Fund Advisors, 2024). The second design featured only 8 rows of circular pins in a wall to wall formation; it also utilized a large number of beads though the exact amount is unknown (Business Performance Improvement, 2024).

These examples supported the idea that both peg shapes could work effectively as a part of the design thus furthering the decision to optimize a prototype for each peg shape ("Galton Board with Pascal's Triangle", n.d.; Business Performance Improvement, 2024). Additionally, they showcased that a great number of beads corresponded with a more accurate distribution (Business Performance Improvement, n.d.; Index Fund Advisors, 2024). Therefore, to test each design accurately, a sufficiently large number of beads was used.

### **Summary & Implications**

There were many key takeaways from literature which aided in the design process. Most importantly, literature agreed that hexagonal and circular pegs could work in a Galton board (Business Performance Improvement, n.d.; Index Fund Advisors, 2024). Therefore, designs for each shape were created and inspired by designs in literature then optimized for functionality. Literature also stated that the peg distance to bead size ratio played a key role in skewing a distribution (Chong, 2021; Mulvhill, n.d.; Seymour, 1996). To avoid this issue when designing, the peg distance was set between all versions of a prototype. If the distribution wasn't normal, the bead size would be changed, so the ratio would change. Additionally, since literature found a large number of beads to be necessary, a sufficiently large number of beads were used in the testing phase of each design. Though, a set amount of beads couldn't be established because of board space and bead size.

## Research Map

**Create an accessible 3D Printable Galton Board**

Signed by: Dominic Thomas

**What skews the distribution of balls within a Galton Board?**

The biggest factor contributing to skew is the ball size to peg distance ratio. As the distance between pegs increases, the number of balls which accumulate in the middle cells increases. This affects the peg design as circular pegs are more likely to allow balls to overly accumulate in the middle cells due to the lack of channels (Chong, 2021). To avoid this issue, the set peg distance will be slightly larger than the diameter of the balls used in the board.

**What are the prevalent Galton board designs currently in literature?**

A Galton board is a mathematical model which utilizes dispersion of beads through pegs to showcase the Central Limit Theorem (Seymour, 1996). Different boards utilized different peg designs to do this; circular and hexagonal pegs are most common (Business Performance Improvement, n.d.; Index Fund Advisors, 2024). Since both are utilized in commercial designs, a design using each peg type will be created then tested.

**Fabricate a shareable Galton Board design for teachers which showcases the Central Limit Theorem**

A successful design will be easily accessible and printable, accurate in portraying a normal distribution, and approved by Dr. Thomas or Dr. Harron

Galton boards designs with circular and hexagonal pegs will be modeled on Fusion 360 then printed and iterated until they run with minimal issue and are approved by the mentors. The distribution of beads for each design will be compared to a normal distribution. The design which best approximates normal will be posted along with assembly instructions on a website shareable with teachers.

## Scope of Study

### Definition of Terms

Key Terms	Definitions
Quincunx	Quincunx was the original name Sir Francis Galton gave for the Galton board (“Galton, Francis, 2020).
Central Limit Theorem	The central limit theorem is a statistical theorem which states that if there is a sufficiently large distribution of random mean sample statistics, the distribution will be approximately normal (LaMorte, 2016).
Normal Distribution	A normal distribution also known as a bell curve describes the shape of a univariate data set when it is symmetric, bell shaped, and has a peak around the mean of the data set (“Measures of Shape”, n.d.).
STL File	STL is an acronym for “stereolithography”. This file format is commonly used for 3D printing since it converts the visual edges of the model to plain text triangles and vertices. It is necessary for creating G-Code files (Adobe, n.d.).
G-Code File	G-Code is an acronym for “geometric code”. This file format is what a printer follows when it is printing as it contains the set of instructions for the printer. It will include things such as printer settings and printing path (Kloski & Kloski, 2022).

### Assumptions

- The Central Limit Theorem is an advanced topic which students struggle to learn.
- Teachers will use visual mathematical aids to help students understand topics.
- Teachers have access to 3D printers and laser cutters.
- Teachers are willing to print and assemble the Galton board design shared with them.
- All teachers will be able to access the final Galton board website.

### Limitations & Delimitations

- This study only included the design process of creating a Galton board; it did not test

whether it increased student comprehension.

- In the Kennesaw State University MAD Lab, there were three printer types available to prototype on: Ultimaker S3s, Ultimaker S5s, and Creality Ender 3s.
- The final website could not be posted to the KSU MAD Lab's server due to KSU permission and access rules.
- Only one design could be tested and iterated each day because of long printing times.
- 4.5 mm, 2.0 mm, 1.8 mm, 1.5 mm, 1.2 mm beads were available to be tested with, in the KSU MAD Lab.

### **Methodology & Design**

The main goal of this project was to create an easily printable and accessible Galton board design for teachers to use in the classroom to showcase the central limit theorem. It was necessary that the model was not only accurate in showing a normal distribution, but also that it was shareable with teachers, so they could print the boards themselves. In order to achieve this, a model was created utilizing a 3D modeling software then iterated until it accurately portrayed a normal distribution. The final design was posted on a website shared with teachers.

### **Tools & Software**

With a goal of designing an easily printable and accessible Galton board for teachers, it was important to establish what modeling software, printer, and coding language would be used. Fusion 360 is a free 3D modeling software for students that can easily and reliably export STL files (Autodesk, 2024). For this reason, each design was made and iterated using this software. When printing, the same printer needed to be used for each prototype. Of the printers available in the MAD Lab, Creality Ender 3s were the most affordable; therefore, it was likely most teachers had access to one (Sink, 2021). Because of this, Creality Ender 3s were used for prototyping each design. In order to make the design available to teachers, the STL file for the 3D model was posted on a website. This website was coded in 3 JS because both professors at the KSU MAD Lab were knowledgeable in it, and it could be used to render a 3D model of the final design into the website.

### **Creating Designs**

As previously mentioned, there were two prevalent Galton board designs found in literature, one with circular pegs and one with hexagonal pegs; therefore, a prototype for each peg design was created. Since fabricating and iterating both prototypes had similar processes, the

process described below was for developing circular pegs; then it was repeated for hexagonal pegs.

The first and most important step when creating this 3D model was creating a base for the model since everything was built off of it. The base size of this design was 100 mm by 200 mm since that size allowed for the board to print comfortably on a Creality's printing plate. Since literature stated that the bead size to peg distance ratio is important, the peg distance needed to be established, so the bead size could be experimented with (Chong, 2021). This distance was 5.5 mm as the goal bead size utilized in the design was 4.5 mm as defined by the MAD Lab professors. The rest of the alpha design features were inspired by the existing designs found in literature (Business Performance Improvement, n.d.; Index Fund Advisors, 2024).

Once the first design was created, the model was printed, so it could be physically tested. To do this, first the design was exported to an STL file. Second, it was uploaded to the cloud printing system available for printing projects in the lab. Third, it was sliced into a G-Code file. Finally, the G-Code file was sent to the printer queue to print overnight.

After the model was printed, a cover was created. It acted as a window to see into the board while it was in use. Using a laser cutter, the cover was cut from clear acrylic, better known as plexiglass, since it was cheap and abundant. The dimensions of this cover were dependent upon the size of the board after it was printed as printing could have caused the walls to be slightly larger than intended.

Finally, before testing, the design was assembled starting with placing at least 70 4.5 mm beads into the 3D printed piece as this was deemed a sufficient amount for this bead size. Then the cover was placed on top of the board. At this point it was all set to be tested.



### **Testing & Iterating Designs**

To test each design, the board was turned upright, so the bead chamber was vertical and the beads were able to fall through the opening into the pegs. This allowed for the beads to accumulate into the cells at the bottom of the board. While testing each model, notes were made and design changes were decided based on the issues observed.

Though many issues could have occurred in the design process, there are a few main issues which were important to address. First, if the beads got stuck between pegs or on the wall of the board, the design of the walls of the board was changed or slightly smaller beads were exchanged and the board was reset and retested. If the beads got stuck in the bead chamber, the design of the chamber was iterated. If the distribution of beads did not resemble a normal distribution, a design or bead size change was made based on the distribution that was seen.

Once notes were written, necessary action was taken to resolve any issues. If the issues were with the design, the model was iterated on Fusion 360 then reprinted and retested. If the issue occurred with the bead size, new beads were put into the board, then it was retested. The iterating and testing phases repeated until a design with little to no issue in the testing phase was seen and mentor approved.

### **Deciding Final Design**

After both prototypes were finalized, the designs were tested against each other, so the most accurate design could be presented to teachers. To decide this, the testing process was run once again. The design which clearly illustrated a better bell-curve shaped distribution was the design set to be shared with teachers.

### **Sharing with Teachers**

The final step in the process of creating a Galton board was making the design available to teachers. This was done by posting the STL file of the final design on a website accessible by teachers along with instructions on how to make it.

First, a folder to hold the website's files was created. Then the node files containing the 3 JS classes and objects were imported into this folder. Next, this folder was opened in VS Code where the base HTML, CSS, and JS files were created. From there, the STL file for the final design was added to the website folder. Using 3 JS classes, the STL file was loaded into the website in 3D. Then, a scene was rendered around it allowing the user to view the board on a plane from any angle and side. This is when the base website was considered done and needed mentor approval.

After completing the base website, a link to a downloadable version of the STL file was added to the webpage. Also linked was a document which contained the assembly instructions. These instructions held information on how to print the model, make the cover, and put everything together. At this point, the website was complete and once again needed to be approved by the mentors. Once that was complete, the website was added to Glitch where it could be viewed by teachers.

### **Validity and Reliability**

Many measures were taken to ensure the validity and reliability of this process. Firstly, designs were consistently reviewed by the same criteria then mentor approved. Secondly, the initial designs were based on current valid designs found in literature (Business Performance Improvement, n.d.; Index Fund Advisors, 2024). Finally, the designs were created on industry recognized software and equipment.

## Results

In order to actually fabricate a Galton board design meeting all of its requirements, three steps needed to be taken. The first two tasks in creating the Galton board design were to create viable prototypes of each peg design which would be compared to each other. The third and final step was to create a website that featured a visual of the final Galton board, a downloadable STL file of the Galton board, and the steps to creating the Galton board.

### **Circular Peg Prototype**

The first prototype created featured circular pegs. For this prototype, the peg distance was set to 5.5 mm as the goal bead size was 4.5 mm, and the base size of this board was 100 mm by 200 mm as this would fit comfortably on the printer bed.

### ***Alpha Design***

The first design of this prototype was informed by the design in literature including circular pegs, and many features from that design were implemented into this design (Business Performance Improvement, 2024). To start, the bead chamber walls were constructed with a straight line connecting the top of the board to the bead chamber opening. This opening was big enough to fit one 4.5 mm bead. The pegs were placed in a triangular formation starting with one peg just below the bead chamber opening and 12 above each cell wall. The side walls were formed with straight lines around this formation. At the bottom of the board, 13 cells were created to catch the beads. Finally, the sides of the board were extruded high enough, so the cover of the board could sit on the board. This design was printed and tested using 4.5 mm beads, and it was observed the beads got stuck in the top chamber of the board, and as shown in table 1, the distribution was a reverse bell-curve.

### *Iterations*

**Version 2.** In version 2, three main changes were made due to issues seen with version 1. Firstly, the peg formation was changed from triangular to wall to wall. Secondly, the tops of the cells were changed from straight across to triangular. Finally, due to the peg formation changing, a funneling system needed to be created to allow beads to collect back into the bead chamber. This was done by creating walls which curved around the pegs. When tested, the greatest issue was beads not falling through the bead chamber opening and getting stuck on the bead chamber side walls. Despite this, the distribution somewhat resembled a bell-curve though it had many outliers.

**Version 3.** In the final version of this Galton board, the opening between the bead chamber and pegs was widened to fit three beads instead of just one, and the funneling system was changed to be more circular. When this final design was tested, the distribution of beads was bell-shaped with few outliers. Though beads still got stuck, it was only minimally with the 4.5 mm beads, and it was fixed if smaller beads were utilized. Additionally, this design was mentor-approved therefore, it was chosen as the final circular peg design.

**Table 1**

#### *Circular Peg Iteration Results*

Version #	Changes	Distribution Shape	Where Do Beads Get Stuck?
1	Initial Design	U-Shaped	Opening
2	Peg formation, Tops of Cells, and Funnel Form	Bell-Shaped with many outliers	Bead chamber
3	Wider opening and Funnel Form	Bell-Shaped with few outliers	Minimally at the opening

**Figure 1**

*Circular Peg Designs in Chronological Order*



### **Hexagonal Peg Prototype**

After the first prototype was optimized, the hexagonal peg prototype was started. This prototype's goal bead size was 1.5 mm as defined by Dr. Harron. Because of the geometry of the hexagons, the peg distance was 3.17 mm. For the same reason, it had a slightly smaller base size of 100 mm by 175 mm. When testing this prototype at least 150 1.5 mm beads were placed into the board as that was a sufficient amount for that bead size.

### ***Alpha Design***

In the initial design, many features from the Galton board found in literature were utilized (“Galton Board with Pascal’s Triangle”, n.d.). First, the pegs were placed in a triangular

formation, but instead of having the bottom row of pegs aligned above each cell wall, they were placed onto the 12 cell walls. Instead of having straight lines for side walls like the other alpha design, the side walls were shaped around each outside peg as shown in figure 2. However, like the first prototype, the bead chamber side walls were created using straight lines. Finally, a cover opening was created, so the cover would slide into the board rather than sit on it. When this design was printed, it was discovered the cover did not fit through this opening, so this design was unable to be tested.

### *Iterations*

**Version 2.** In this version, the position and dimensions of the cover opening were changed. It was realized that having the cover opening at the bottom of the board would lead to beads easily falling out of the board during use; therefore, the cover opening was moved to the top of the board. Additionally, the width of the opening was increased, although this wasn't enough because when tested the cover fit but tightly. As seen in table 2, the distribution of beads modeled a normal distribution well, but beads got stuck in the bead chamber opening.

**Version 3.** Since the cover still didn't fit smoothly into the board in version 2, the width was increased one final time. Furthermore, the bead chamber opening was widened to fit more beads. This was done by removing the sharp lips in the opening seen in versions 1 and 2 in figure 2. The distribution remained unchanged when tested, and the beads didn't get stuck in the opening. Though it was noticed, when the board was flipped upside down, the beads got stuck inside the cells. The cover opening was also slightly too short, as the top lip holding it in would bend to fit the cover.

**Version 4.** For the 4th version, two issues needed to be addressed. Firstly, the cover opening height was increased slightly. Secondly, a wall that acted as a ramp was added between

the cell wall and the side of the hexagon at the top of the wall; this can be seen in figure 3.

Although it was not seen as an issue, the bead chamber was also widened to increase the number of beads which it could hold. This was done by decreasing the slopes of the side walls in the bead chamber. When tested, beads got stuck in the bead chamber again due to the decreased slope. In addition, it was observed the beads got stuck between the outside pegs and side walls.

**Version 5.** Only two changes were made from version 4 to the final version. Firstly, the bead chamber size change made to the last design was undone, so the slope of the walls in the bead chamber were increased. Secondly, the shape of the side walls was changed which can be seen in figure 4. These changes proved very beneficial as when tested, the distribution remained normal, the beads got minimally stuck on the side walls, and the cover fit well into the opening. Because of this, version 5 of this prototype was mentor approved.

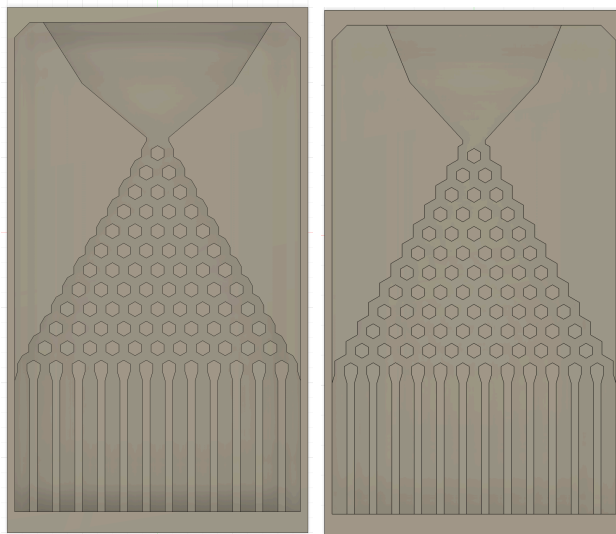
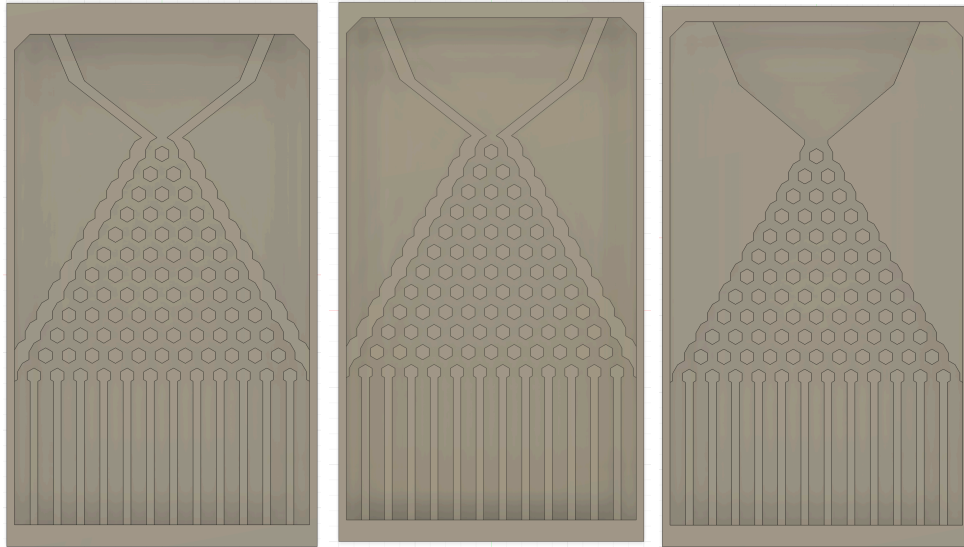
**Table 2**

*Hexagonal Peg Iterations Results*

Version #	Changes	Distribution Shape	Where Do Beads Get Stuck?
1	Initial design	Unable to be tested	Unable to be tested
2	Cover opening position and dimensions	Bell-shaped	Bead Chamber Opening
3	Cover and bead chamber opening	Bell-shaped	Cells
4	Cell wall shape and bead chamber	Bell-shaped	Bead Chamber and Side walls
5	Side walls and bead chamber	Bell-shaped	Minimally on side walls

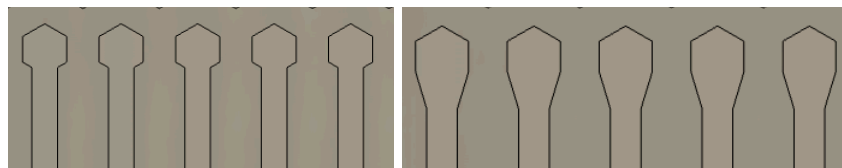
**Figure 2**

*Hexagonal Peg Designs in Chronological Order*



**Figure 3**

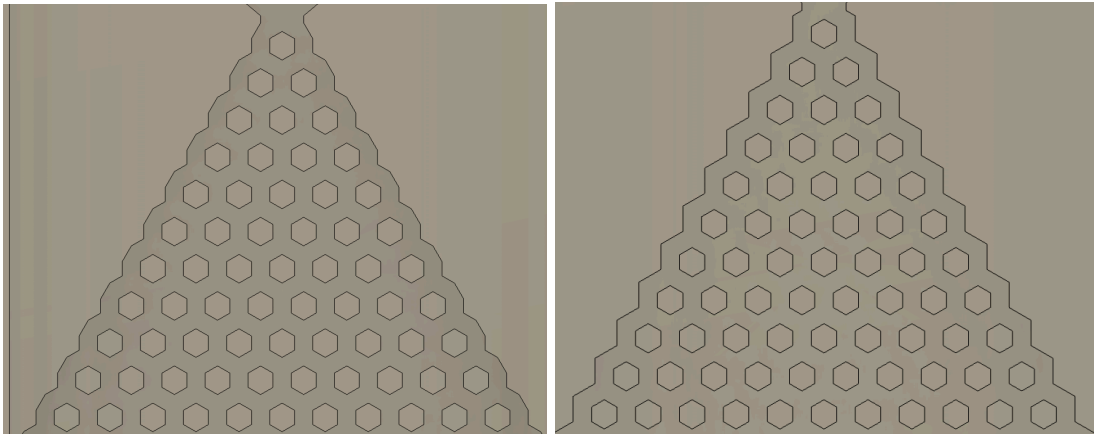
*Cell wall designs: old design on the left and new design on the right*





**Figure 4**

*Side wall designs: old design on the left and new design on the right*



### **Website to Share Design**

This project aimed to create a Galton board design which teachers could access; therefore, the final step in this project was creating a website for teachers. The first step in creating this website was making a base to hold each of the files outlined in the methodology. For this project, the base website had a title and a visual of the final design. The final website included these features as well as the assembly guide and downloadable Galton board STL file.

#### ***Base Website***

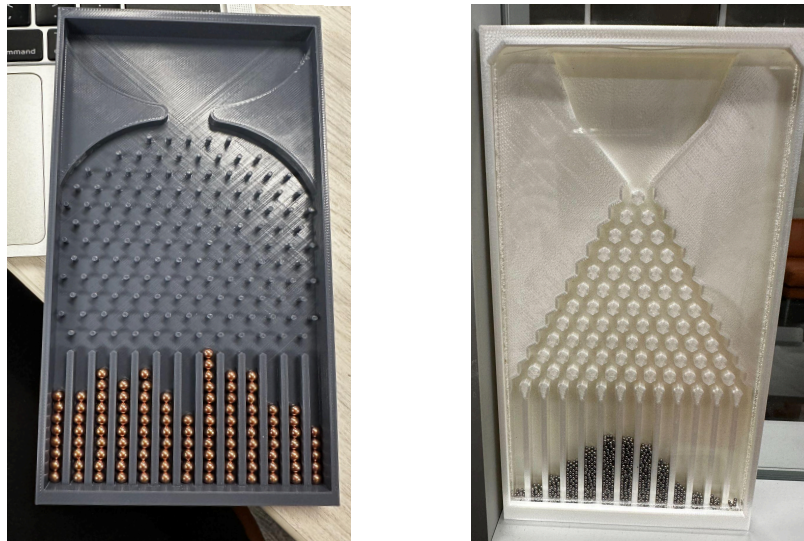
The first feature to be implemented into the website was a visual of the final Galton board design. To implement this, a final design needed to be decided. Using the process outlined in the methodology, it was decided that the hexagonal peg prototype was going to be the final design shared with teachers as its distribution was much more bell-shaped as shown in figure 5.

Since this website was coded in 3 JS, the STL Loader class of 3 JS was used to load the final hexagonal Galton board into the website. Once that was loaded into the website, the cube geometry class was used to add a virtual cover to the board. Figure 6 shows what the website

looked like when the STL file and cover were added. These served as the visual of the final design for teachers. A scene was then rendered around this visual using other 3 JS classes. Finally, using the Orbital Controls class, zooming, rotating, and panning around the board were added. At that point, the base website was considered done and was mentor approved.

### Figure 5

*Distributions of final circular peg design and final hexagonal peg design*



### Figure 6

*Website with Galton board visual*



### *Final Website*

Three different files were necessary components of the website to conclude it was done. As previously stated, the first two were the STL file of the final design and the assembly guide. The 3rd file was the SVG file of the cover, so teachers could have the correct dimensions to laser cut the cover to.

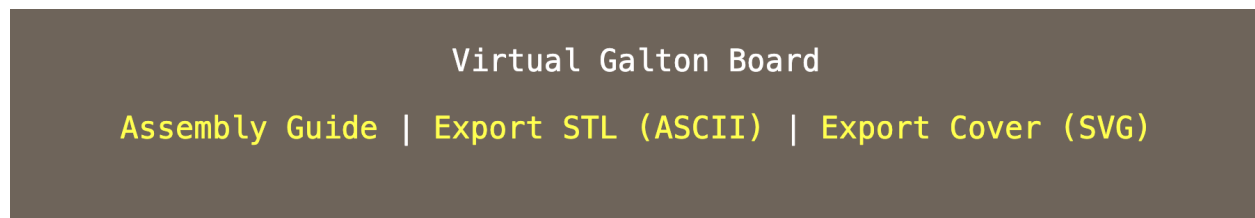
**STL and SVG Files.** Since they were the easiest, the first files to be implemented into the website were the final STL and SVG files. To implement these, a simple link to the file with the download attribute was added to the header of the page using HTML.

**Assembly Guide.** To implement the assembly guide, a text document was created and split into 3 sections: printing the board, cutting the cover, and assembling. In each section an in depth step by step guide including applications and settings was written. Once the guide was created, the link was added to the header of the page as shown in figure 7.

Once each of these features was implemented, the website was mentor-approved and considered done, so it was added to Glitch where teachers would be able to access it. The final website can be seen in figure 8.

### **Figure 7**

#### *Header of the Website*



**Figure 8**

*Final Website with every feature*



### Conclusions

As previously stated, the main goal of the project was to create a printable Galton Board for teachers to utilize in the classroom. This was accomplished by analyzing current literature on Galton boards, creating and optimizing a design, and displaying the design on a website. From literature, it was found that two different peg shapes are popular in Galton boards, circular and hexagonal. Thus two different prototypes were created, one with circular pegs and the other with hexagonal pegs. Further, literature stated that peg distance to bead size ratio affects the shape of the distribution of beads (Chong, 2021).

With this in mind, a design for each prototype was created and optimized. The alpha designs for each prototype were informed by Galton boards that already existed in literature (“Galton Board with Pascal’s Triangle”, n.d.; Index Fund Advisors, 2024). Each design was 3D printed, tested, and iterated until it was functional. Once this was done, the prototypes were tested against each other. The prototype featuring hexagonal pegs was found to create the most normal-shaped distribution; therefore, it was declared the final design to be shared. After that, a website was created to host the final design. This website held files such as the final STL file of the board, SVG template for the cover, and assembly guide for the board. Additionally, it showcased a visual of the final Galton board according to how it was created in the assembly guide. Once complete, this website was posted on Glitch, so teachers could access it.

Officially, the criteria for the Galton board design was that it needed to be accessible, easily printable, and accurate as well as mentor-approved. In this project, each part of this criteria was addressed. First, with the creation of the website, the Galton board could be considered accessible because teachers can utilize it. Second, since a guide to creating a Galton board was added to the website, the design could be easily printable for teachers who may not be

experienced in 3D printing. Third, since the board was optimized for creating a normal distribution through testing and iteration, it was accurate. Finally, both the website and final prototypes were mentor-approved. Thus, this project was deemed successful for addressing all of the design requirements.

Surprisingly, the results of this project slightly disagreed with literature. In literature, it was accepted that both peg shapes can work in a Galton board which was concluded in this project as well. However, the hexagonal peg shape worked much better which was not shown in literature. Nevertheless, this project agreed that the bead distance to peg size ratio is an important factor in the Galton board.

### **Limitations and Improvements**

Many assumptions were made regarding this project, one of the biggest being that all teachers have access to 3D printers and laser cutters. This assumption limits the number of teachers who can access this design because teachers who may not have access to those tools are excluded. Additionally, since the website could only be posted to Glitch, the final website might be difficult for teachers to find or open because of the bandwidth limitations of Glitch. This also limits the amount of teachers who can access the board as if teachers can access the website, they can't utilize the design.

Additionally, despite meeting all of the design requirements, the actual final Galton board design could be improved. First, as shown in Table 2 the beads did not flow smoothly through the board as even though minimally, they got caught between the sidewalls and outside pegs. Additionally, if the board was held at even a slightly sideways angle, the distribution of beads would be skewed due to gravity.

**Future Work**

In the future, the limitations and flaws of this design would be addressed. First, the hexagonal prototype design would be further iterated and tested until it runs smooth, and the issue of beads getting stuck between the outside pegs and side walls would be eliminated. Then, a stand would be added to the back of the board so a proper upright position could be established when running the board. As well, since there may be teachers who don't have access to 3D printing, a simulation of the final design could be displayed on the website for teachers to show their students in class. Finally, if possible a way to implement the final website folder onto the MAD Lab website would be worked out, so it could be more accessible.

**Implications**

With what was done, this project had a profound impact in and beyond the KSU MAD Lab. In the MAD LAB, the Galton board could be displayed for students to pick up and utilize as a learning opportunity. In the KSU statistics department, this tool is likely to be implemented in the classroom for student use. In literature, this project contributed to the small amount of papers written on the fabrication of Galton boards. For other researchers, because the process of its creation was detailed herein with files available for download, this design could be furthered and modified to fit applications beyond the central limit theorem. For teachers across the globe, this tool added to the ever-growing pool of visual mathematical tools used to teach students.

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